ECE317 : Feedback and Control

Lecture :
Frequency domain specifications
Frequency response shaping (Loop shaping)

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Course roadmap

Modeling

- Laplace transform
- Transfer function
- Block Diagram
- Linearization
- Models for systems
  - electrical
  - mechanical
  - example system

Analysis

- Stability
  - Pole locations
  - Routh-Hurwitz
- Time response
  - Transient
  - Steady state (error)
- Frequency response
  - Bode plot

Design

- Design specs
- Frequency domain
- Bode plot
- Compensation
- Design examples

Matlab & PECS simulations & laboratories

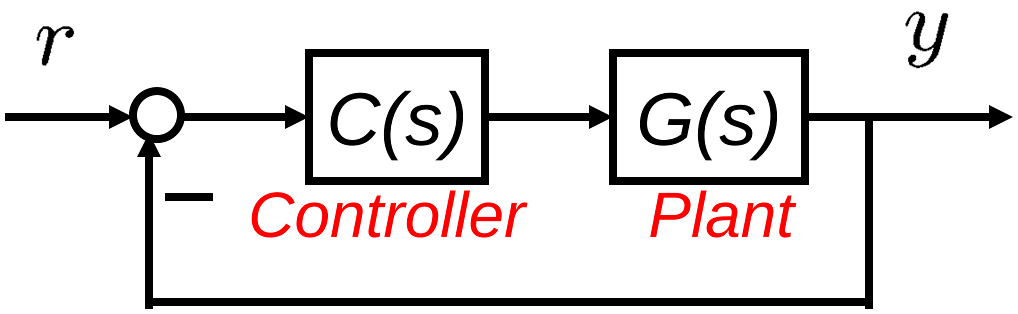
Controller design comparison

Design specifications in time domain
(Rise time, settling time, overshoot, steady state error, etc.)

Approximate translation

Desired closed-loop pole location in *s*-domain	Desired open-loop frequency response in *s*-domain
Root locus shaping	***Frequency response shaping (Loop shaping)***

Feedback control system design



OL: $L(s) := G(s)C(s)$

CL: $T(s) := \dfrac{L(s)}{1 + L(s)}$

- Given *G(s)*, design *C(s)* that satisfies CL stability and time-domain specs, i.e., transient and steady-state responses.
- We learn typical qualitative relationships between open-loop Bode plot and closed-loop properties such as stability and time-domain responses.

An advantage of Bode plot (review)

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
    - Gain

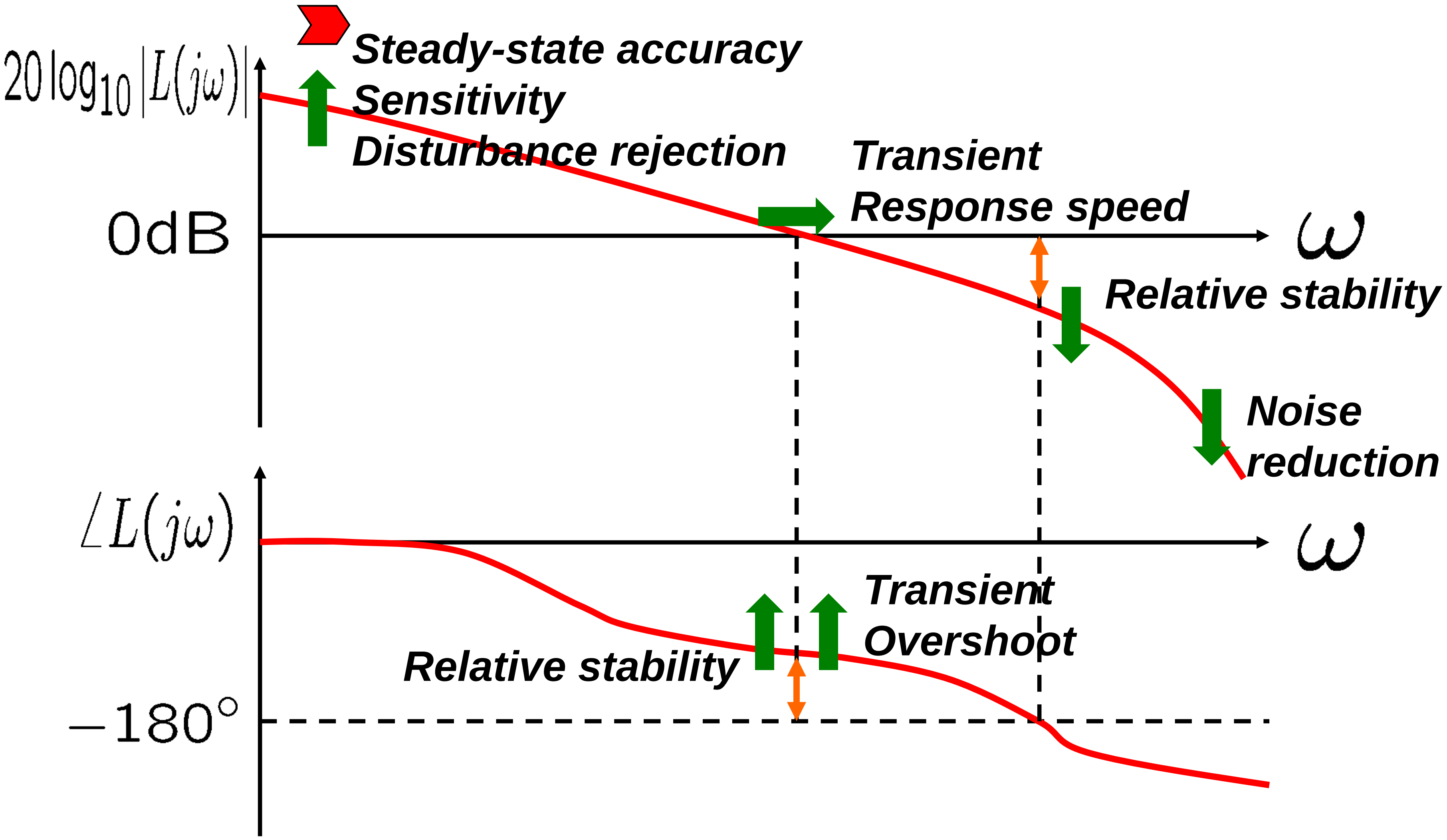
$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

    - Phase

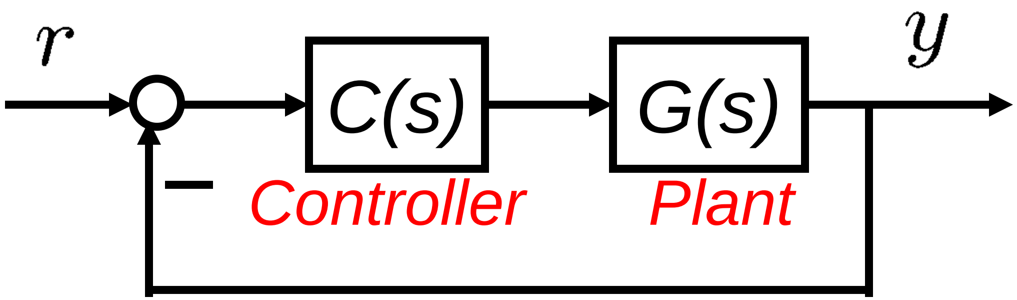
$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- We use this property to design $C(s)$ so that $G(s)C(s)$ has a “desired” shape of Bode plot.

Typical modification of OL Bode plot



Steady-state accuracy

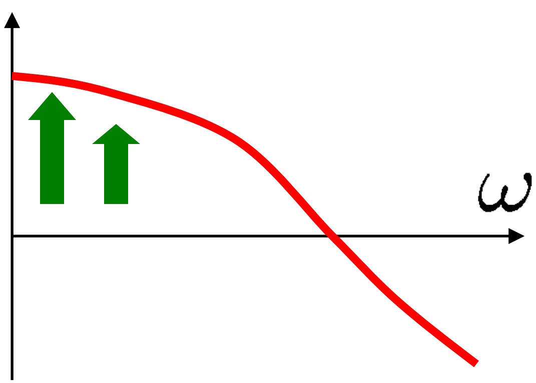


$$L(s) := G(s)C(s)$$

$$T(s) := \frac{L(s)}{1+L(s)}$$

For steady-state accuracy,
L should have high gain at low frequencies.

$20\log_{10}|L(j\omega)|$



large $|L(j\omega)|$

$$\Rightarrow \quad T(j\omega) = \frac{L(j\omega)}{1+L(j\omega)} \approx 1$$

$\Rightarrow$ *y(t)* tracks *r(t)* composed of low frequencies very well.

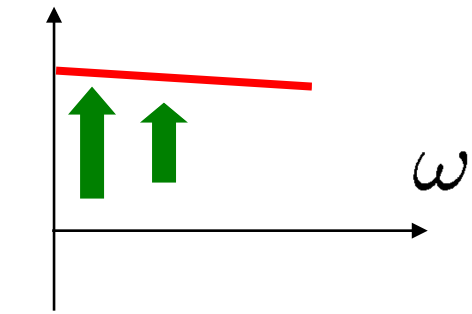
Steady-state accuracy (cont'd)

- Step *r(t)*

Increase

$$K_p := L(0)$$

$$20 \log_{10} |L(j\omega)|$$

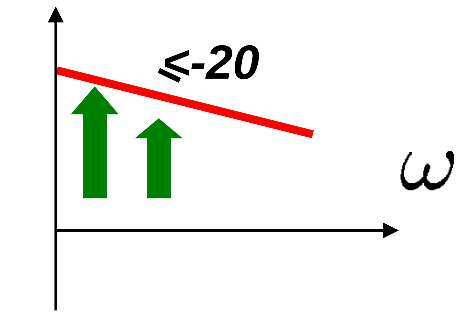


- Ramp *r(t)*

Increase

$$K_v := \lim_{s \to 0} sL(s)$$

$$20 \log_{10} |L(j\omega)|$$



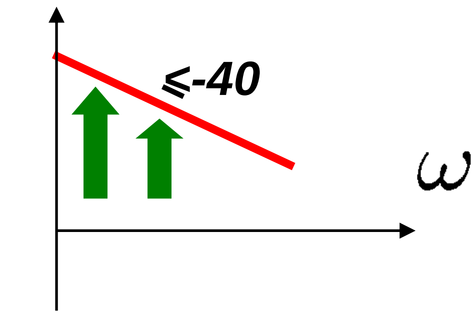
For *Kv* to be finite, *L* must contain at least one integrator.

- Parabolic *r(t)*

Increase

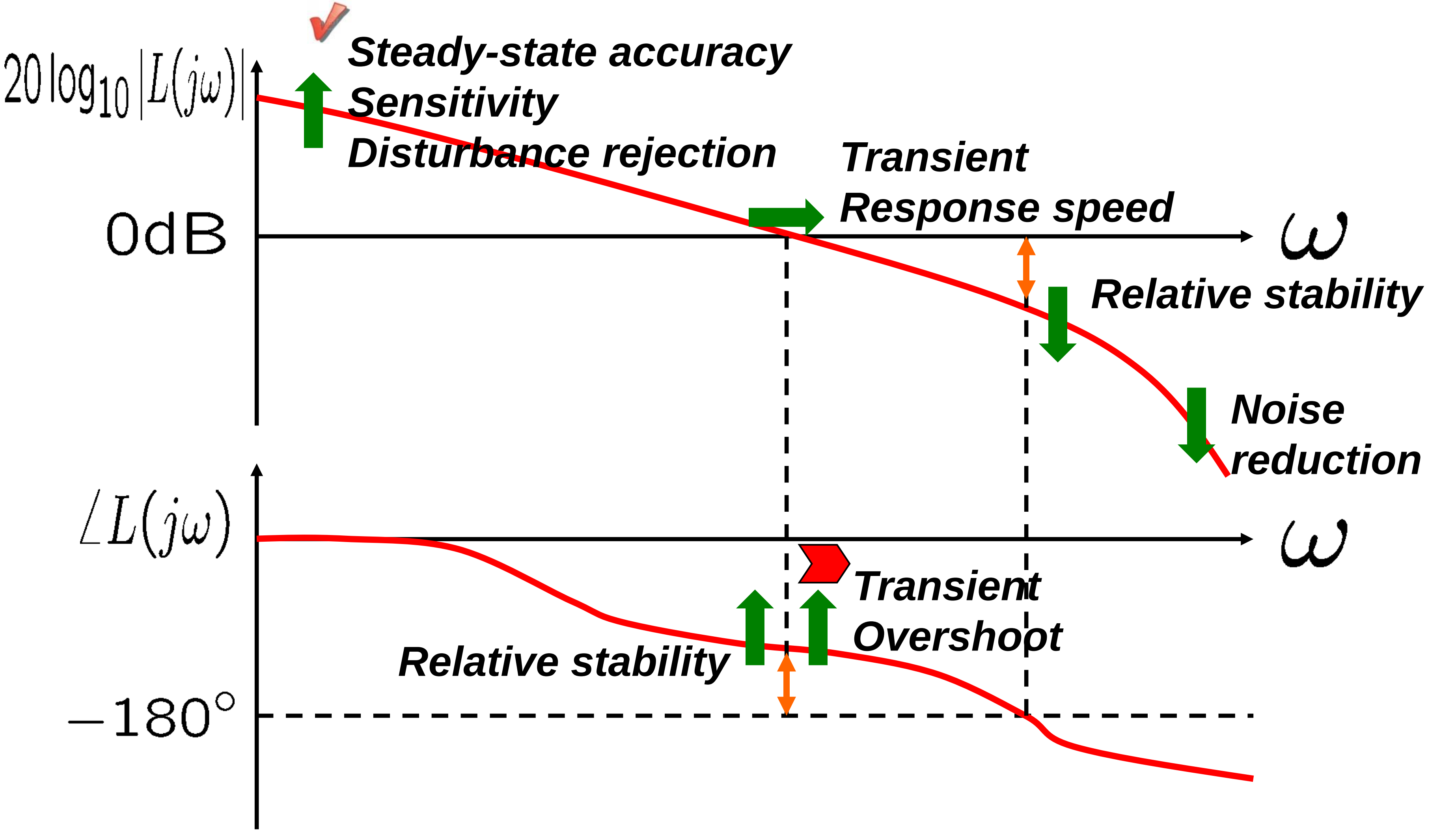
$$K_a := \lim_{s \to 0} s^2 L(s)$$

$$20 \log_{10} |L(j\omega)|$$



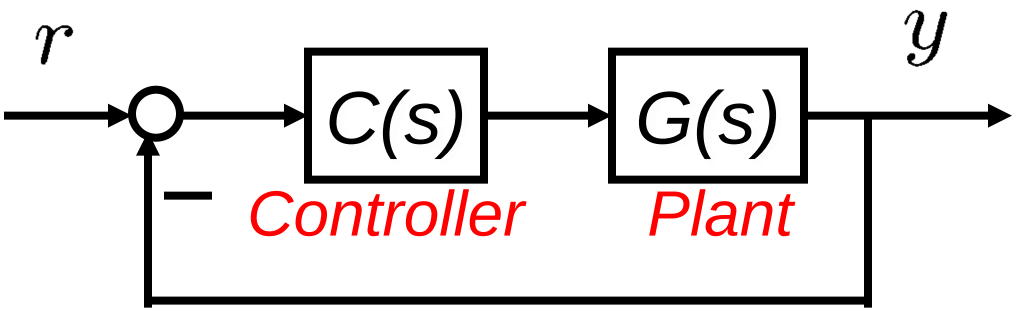
For *Ka* to be finite, *L* must contain at least two integrators.

Typical modification of OL Bode plot



A second order example

- For illustration, we use the feedback system:



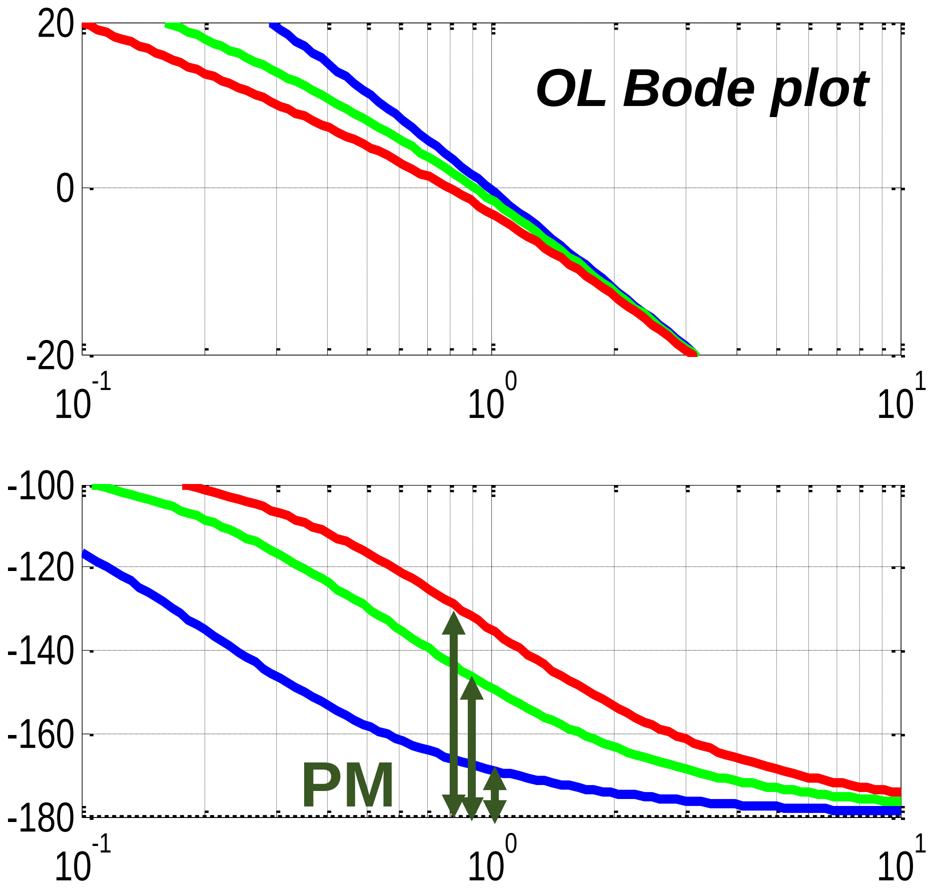
$$L(s) := G(s)C(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

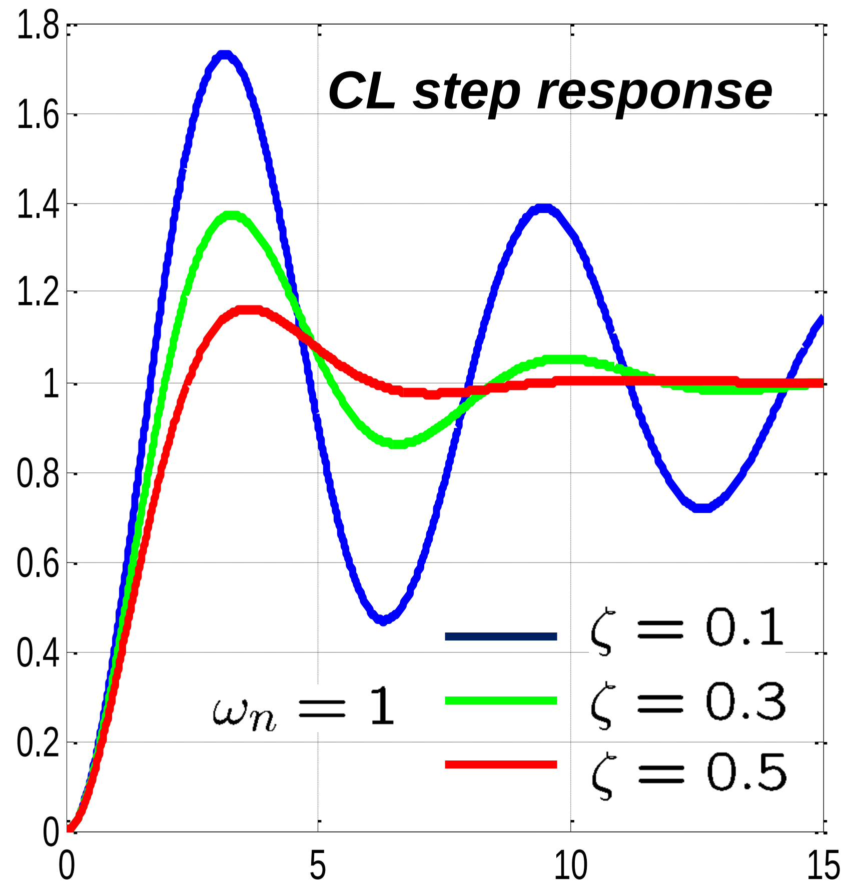
$$T(s) := \frac{L(s)}{1 + L(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Percent overshoot

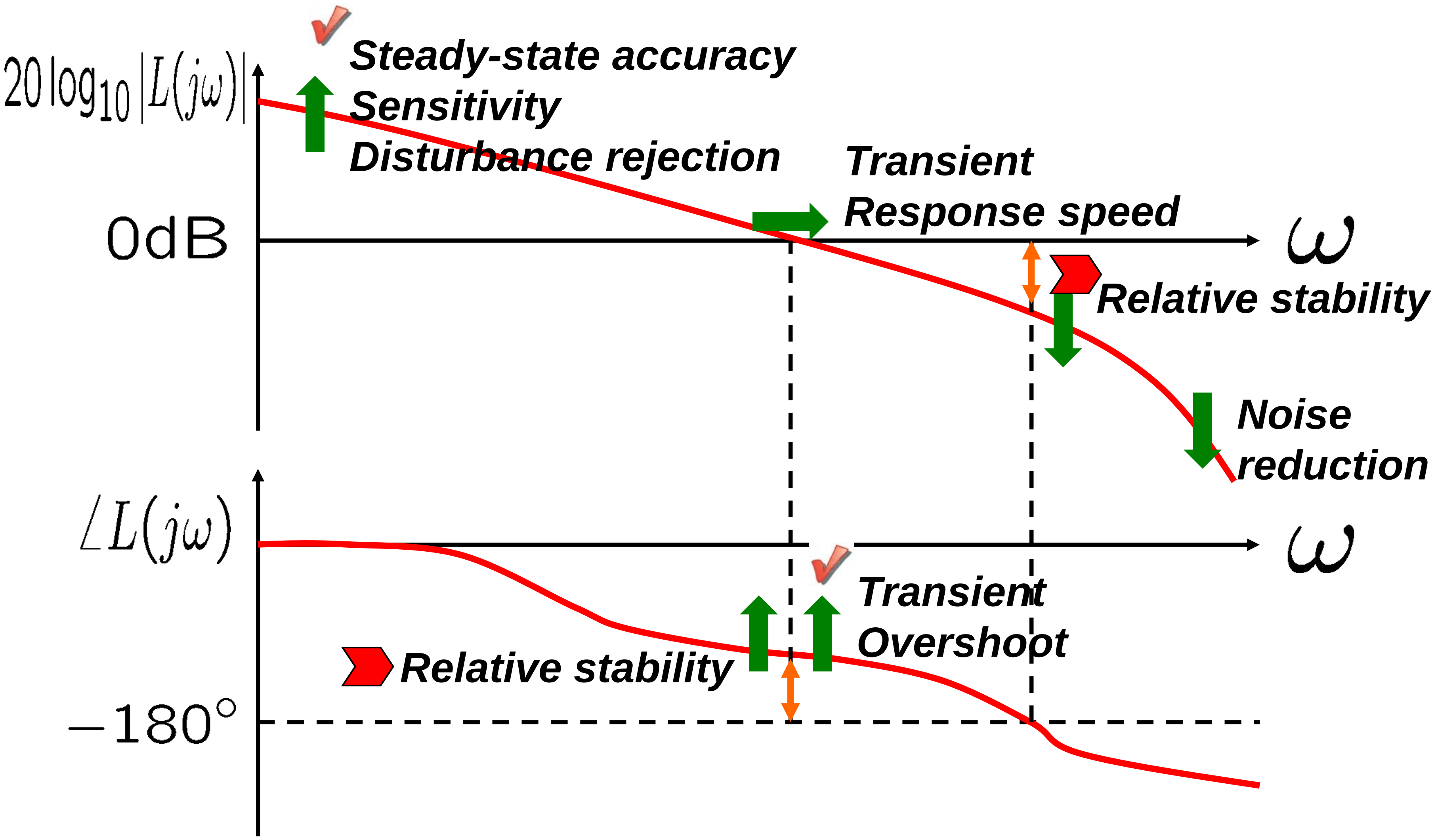
For small percent overshoot,
L should have larger phase margin.

$$\zeta \approx \frac{\mathrm{PM}}{100} \quad \text{for } \zeta < 0.7$$





Typical modification of OL Bode plot

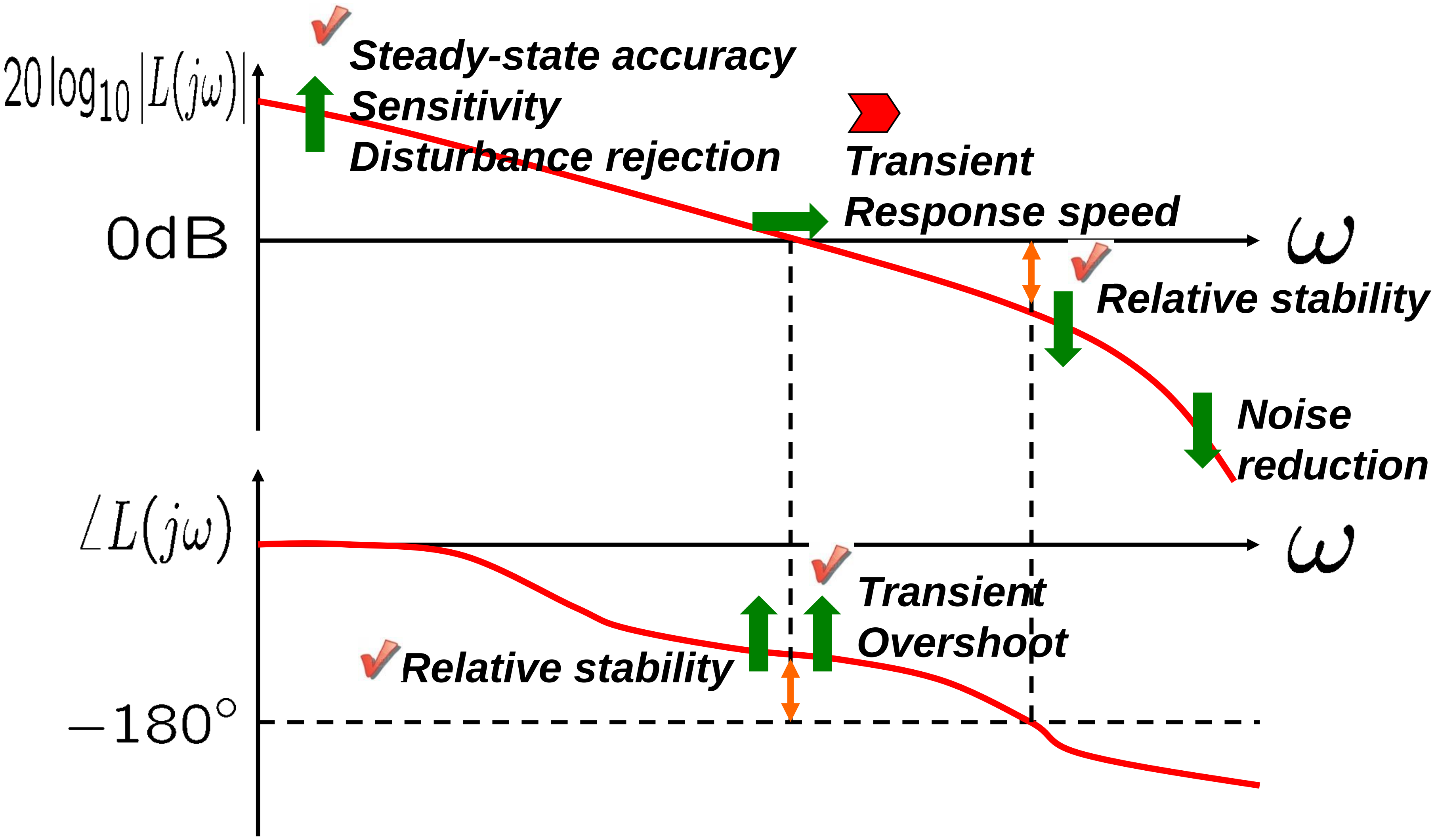


Relative stability

- We require adequate GM and PM for:
  - safety against inaccuracies in modeling
  - reasonable transient response (overshoot)
- It is difficult to give reasonable numbers of GM and PM for general cases, but usually,
  - GM should be at least 6dB
  - PM should be at least 45deg

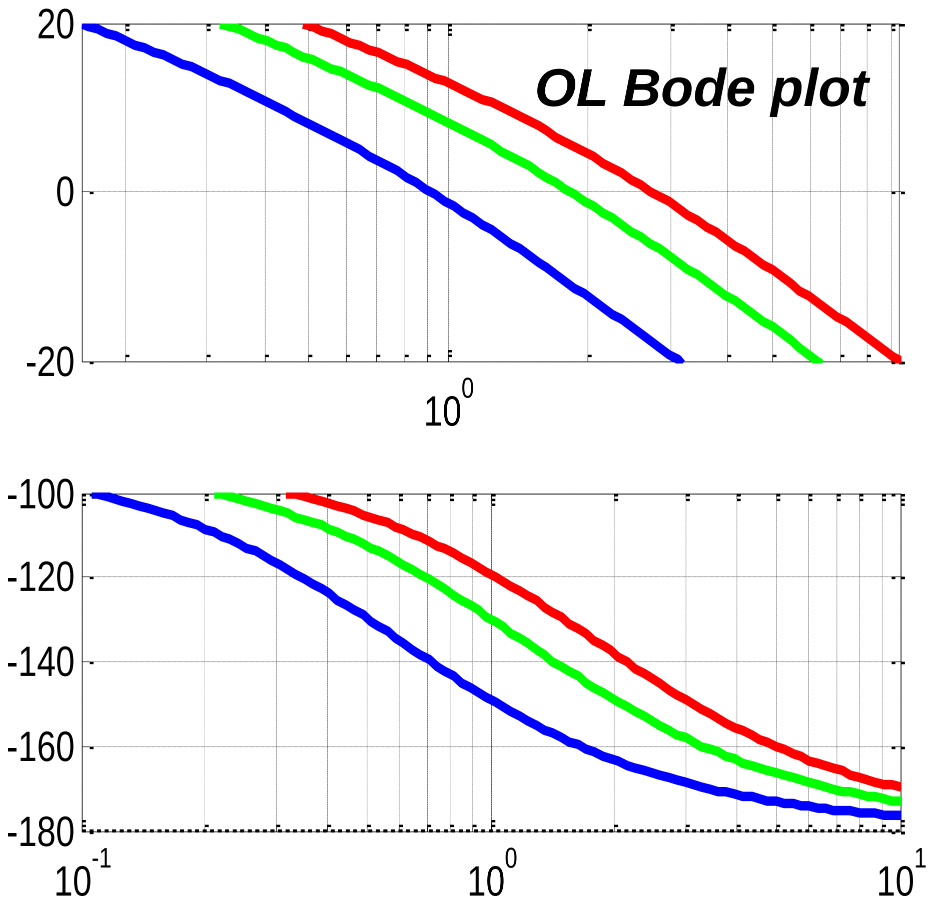
  (These values are not absolute but approximate!)
- In controller design, we are especially interested in PM (which typically leads to good GM).

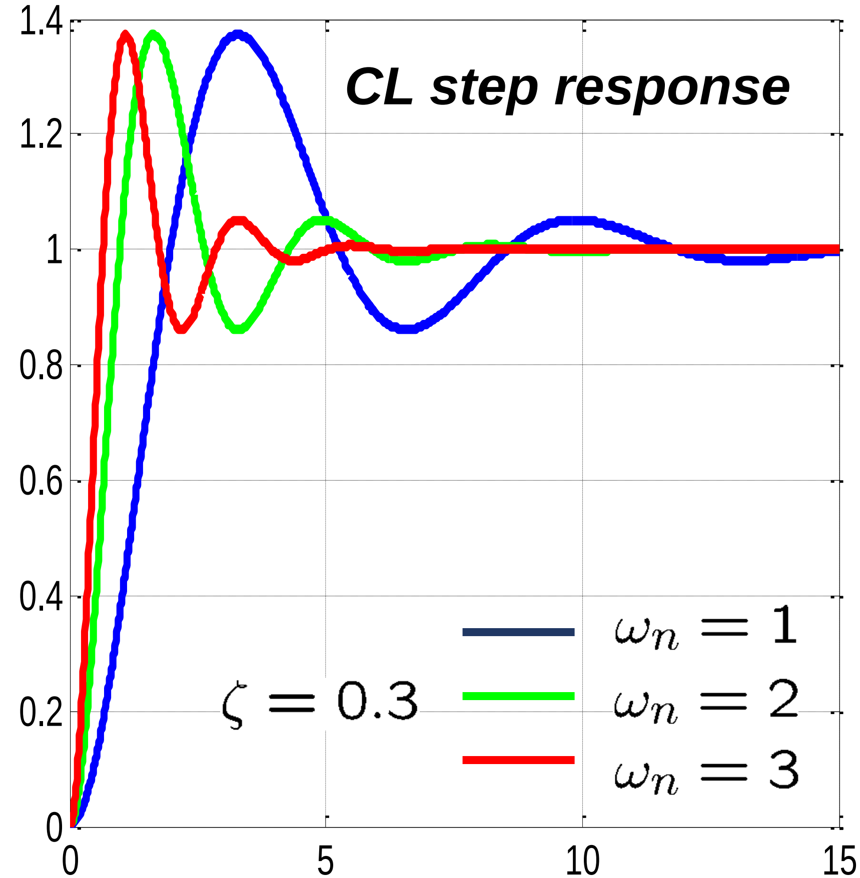
Typical modification of OL Bode plot



Response speed

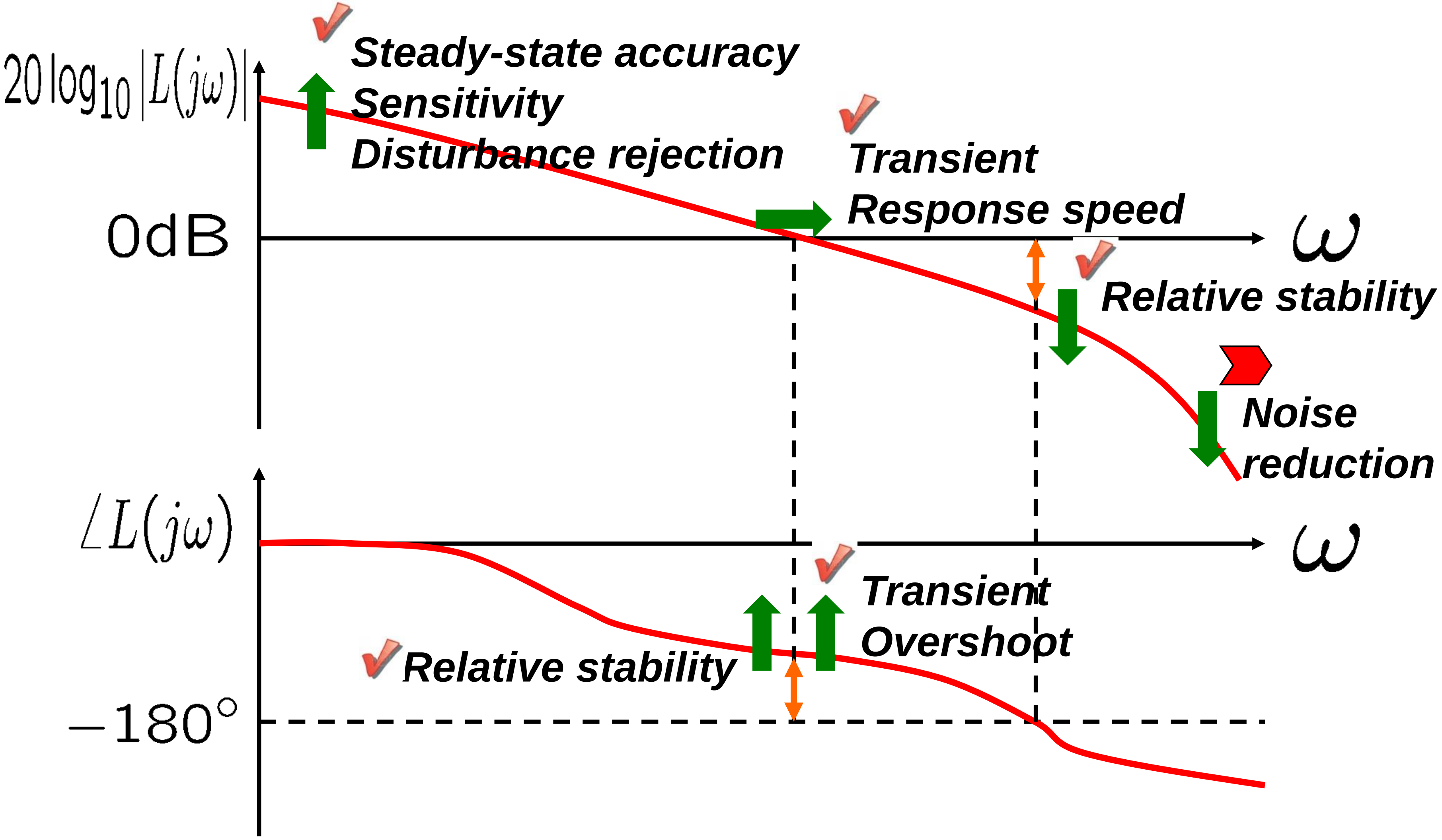
For fast response,
L should have larger gain crossover frequency.



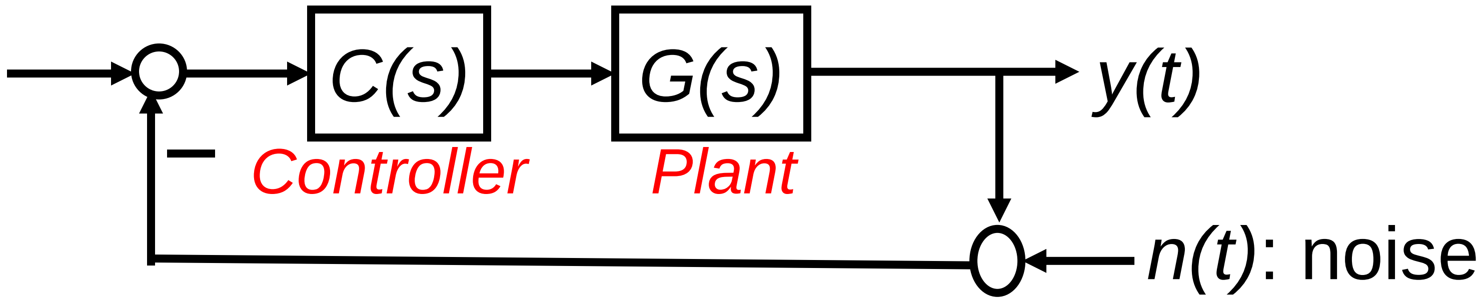


$\omega_g T_r \approx \text{constant}$

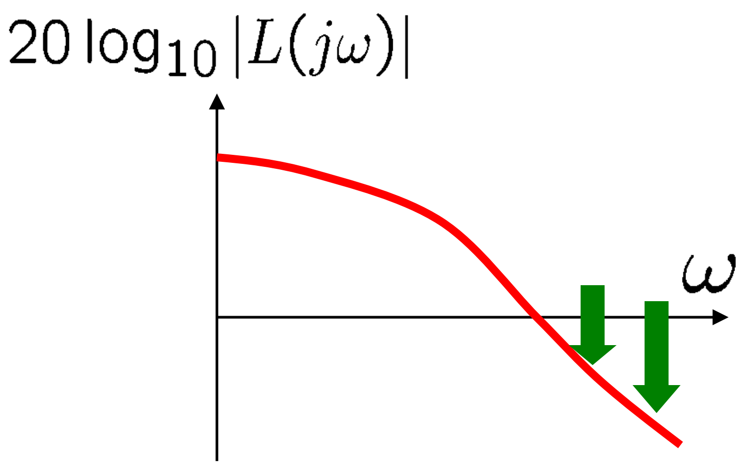
Typical modification of OL Bode plot



Noise reduction



For noise reduction,
L should have small gain at high frequencies.

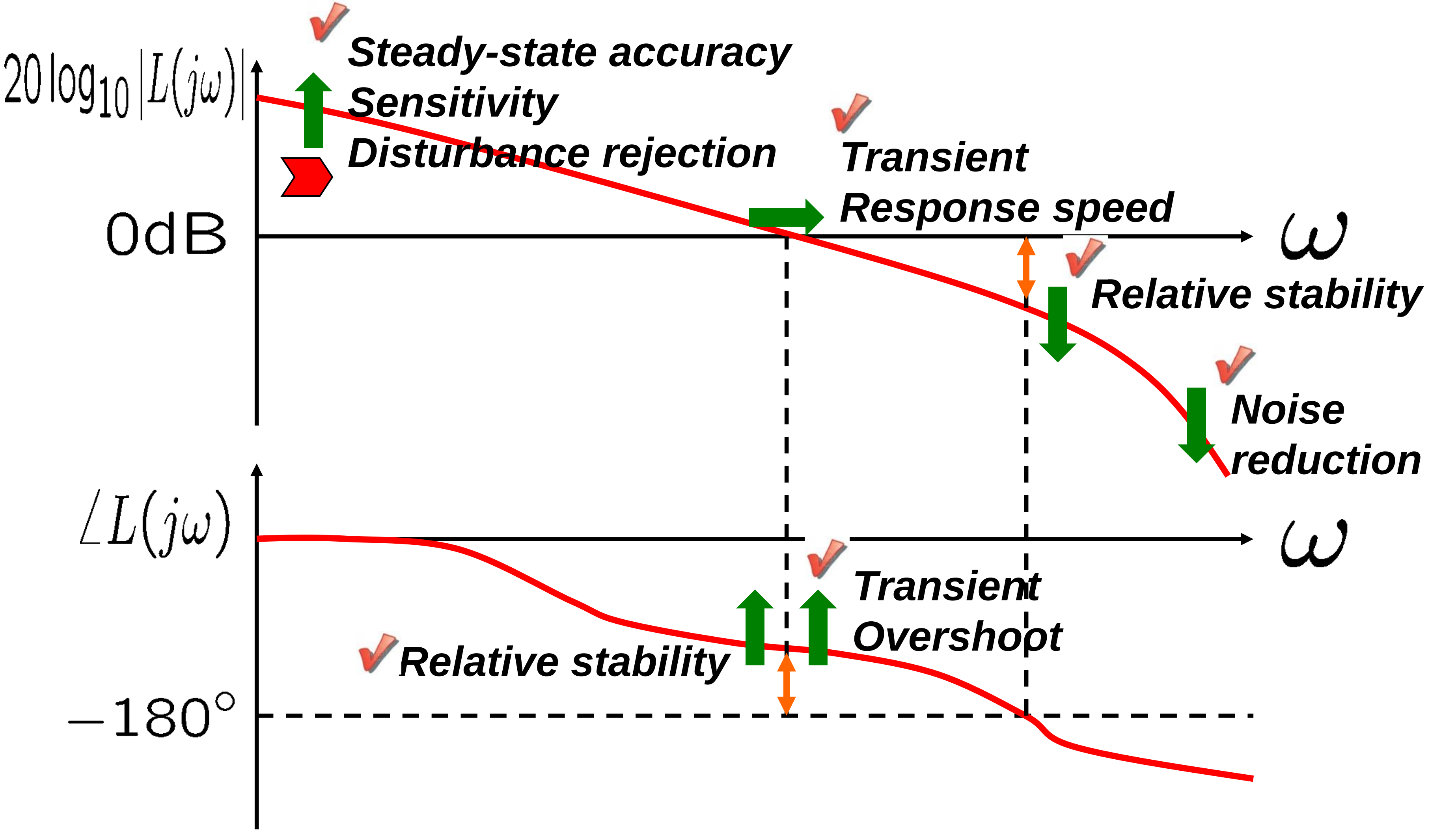


small $|L(j\omega)|$

$$\Rightarrow \quad \frac{Y}{N}(j\omega) = -\frac{L(j\omega)}{1+L(j\omega)} \approx 0$$

$\Rightarrow$ *y(t)* is not affected by *n(t)* composed of high frequencies.

Typical modification of OL Bode plot

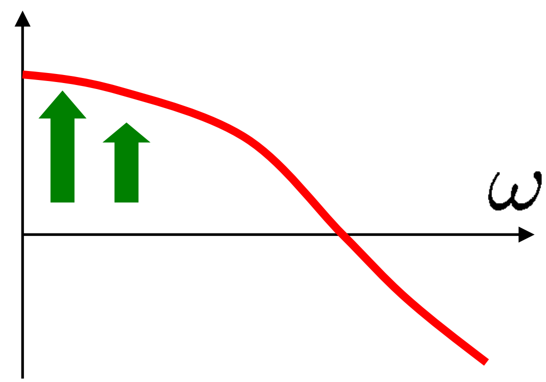


Sensitivity reduction

- Sensitivity indicates the influence of plant variations (due to temperature, humidity, age, etc.) on closed-loop performance.
- Sensitivity function

$$S(s) := \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{1}{1+G(s)C(s)} = \frac{1}{1+L(s)}$$

For sensitivity reduction, L should have large gain at low frequencies.

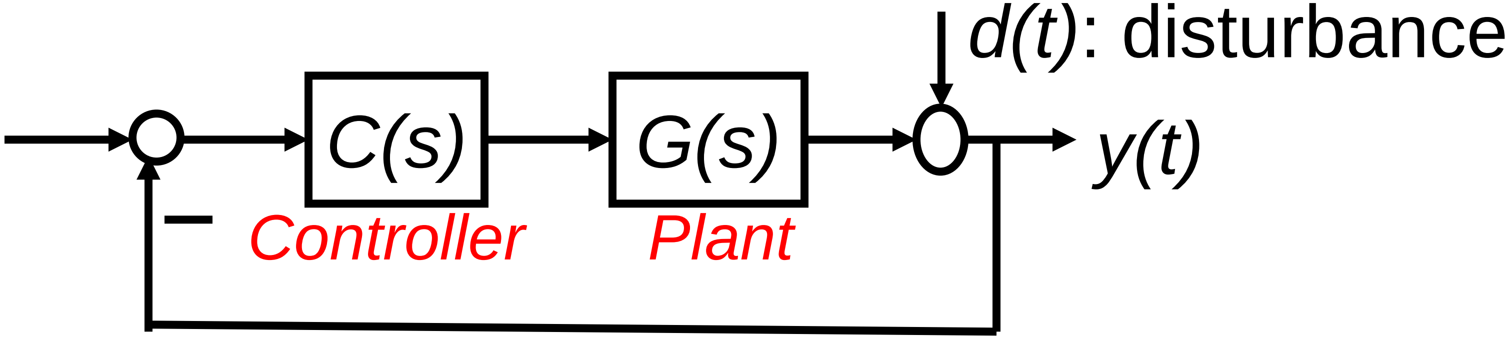


large $|L(j\omega)|$ ⟹ $S(j\omega) = \dfrac{1}{1+L(j\omega)} \approx 0$

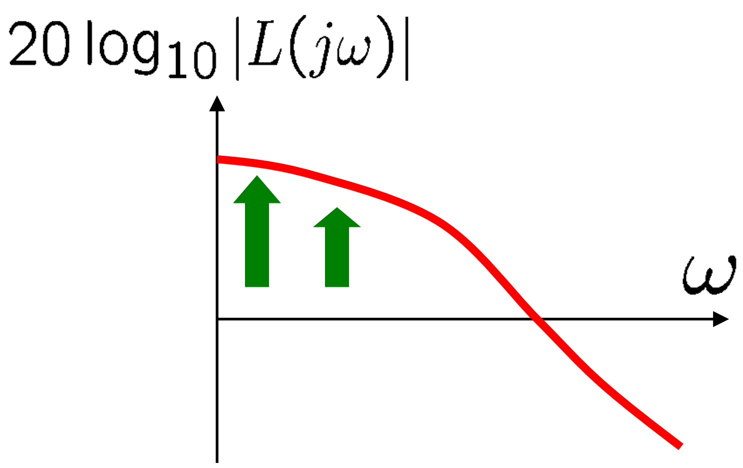
Disturbance

- Unwanted signal
- Examples
  - Load changes to a voltage regulator
  - Wind turbulence in airplane altitude control
  - Wave in ship direction control
  - Sudden temperature change outside the temperature-controlled room
  - Bumpy road in cruise control
- Often, disturbance is neither measurable nor predictable. (Use feedback to compensate for it!)

Disturbance rejection



For disturbance rejection,
L should have large gain at low frequencies.

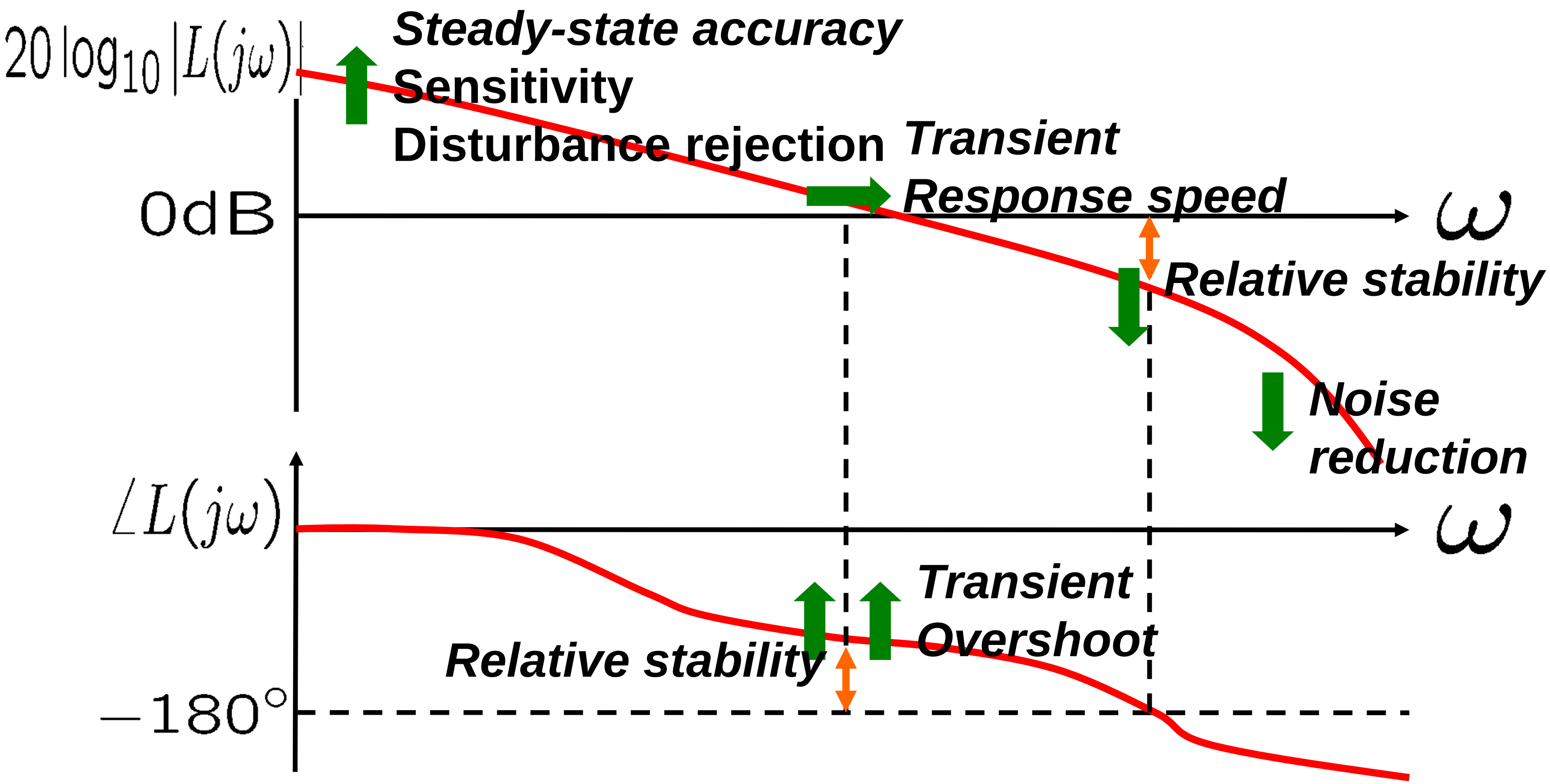


large $|L(j\omega)|$

$$\frac{Y}{D}(j\omega) = \frac{1}{1+L(j\omega)} \approx 0$$

y(t) is not affected by *d(t)* composed of low frequencies.

Typical shaping goal (Summary)



- *Frequency shaping (loop shaping) design is done using compensators*

Summary

- System performance such as transient response and steady state error (time domain attributes) and sensitivity to plant variations and disturbance rejection are addressed by appropriate design in the frequency domain.
- This leads to a set of frequency domain specifications.
- These are specifications are on the loop gain, specifically, low frequency gain, bandwidth (unity gain crossover), phase margin and high frequency roll off.

- Next, steady state error.